— Design of Correct Circuits, Budapest 2008 —

Defining Elastic Circuits with Negative Delays

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Latency-Insensitive Design

- Implement a given functionality in a way that tolerates the latency changes of components and wires connecting them.
- Pioneering work: Carloni, McMillan, Sangiovanni-Vincentelli [CAV'99]
- Intel project **SELF** (Synchronous Elastic Flow)
  - Kishinevsky, Cortadella, . . . [TAU'05, DAC'06, DAC'07]
- Theoretical foundation for simple SELF [FMCAD'06]
- This presentation: Challenge in extending the theory

The Idea of an Elastic Circuit

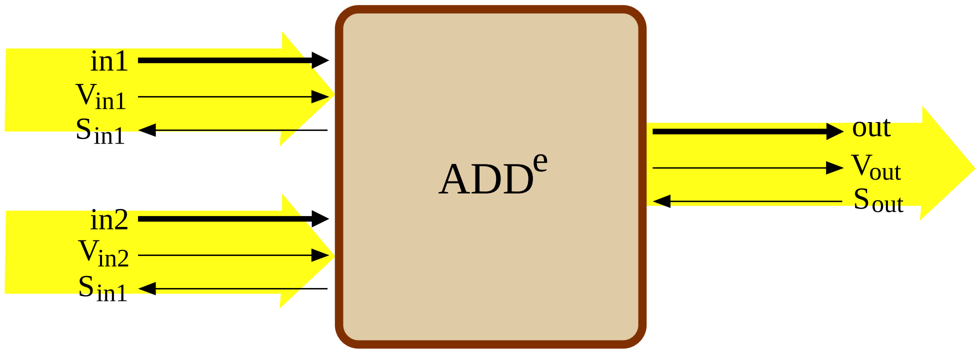
- A behavior of an elastic inverter:

  - data transfer does not occur at every cycle
  - tokens and bubbles

- Why bubbles?
  - valid **input** not there, or INV cannot take it
  - **output** not yet computed, or the consumer not ready to take it

Elastic Control

- Transfer along a wire X occurs when both parties are ready
  - Producer ready: the **valid bit** V_X is 1
  - Consumer ready: the **stop bit** S_X is 0
  - **elastic channels**: wire triples $\langle X, V_X, S_X \rangle$



- Transfer and token count on a channel:

cycle	0	1	2	3	4	5	6	7	8	9	...
X	$*$	A	B	B	B	C	$*$	$*$	D	D	...
V_X	0	1	1	1	1	1	0	0	1	1	...
S_X	0	0	1	1	0	0	0	1	1	0	...
tct_X	0	1	1	1	2	3	3	3	3	4	...

Correctness: Persistence

- If you're ready for handshake, you stay ready till it happens

  - $\Box\,(\mathsf{V}_Y \wedge \mathsf{S}_Y \Rightarrow (\mathsf{V}_Y)^+)$ for every output Y

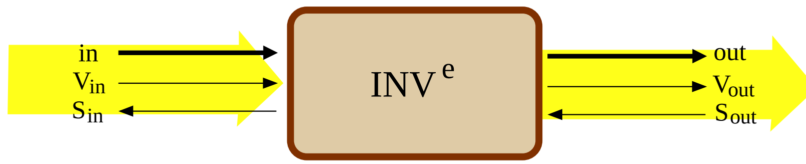
  - $\Box\,(\neg\mathsf{V}_X \wedge \neg\mathsf{S}_X \Rightarrow (\neg\mathsf{S}_X)^+)$ for every input X

Correctness: Data

- Every elastic circuit E is tied to a reference system R
  - E is an **elasticization** of R
- Require: **if you take any behavior of E and ignore the bubbles, the result is a behavior of R**
  - We need to be more precise if we want this property to be proved or model-checked

Correctness: Liveness

- Need a condition strong enough to guarantee infinite flow of tokens on all channels assuming a cooperating environment



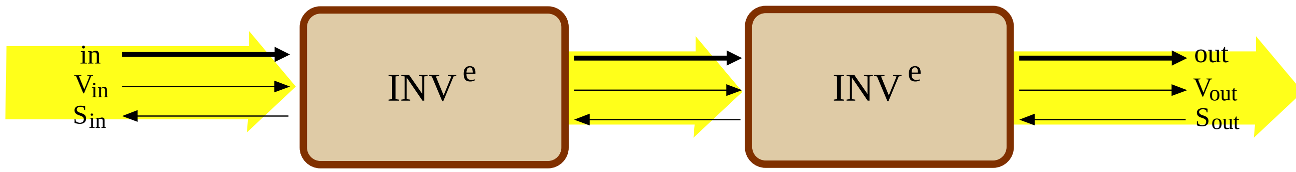
- Weak fairness:
  - $\Box(\Box \text{en\_out}_Y \rightarrow \Diamond V_Y)$ for every output Y
  - $\Box(\Box \text{en\_in}_X \rightarrow \Diamond \neg S_X)$ for every input X

  where
  - $\text{en\_out}_Y = (\text{tct}_Y - \text{tct}_X > 0)$ (there is a token inside)
  - $\text{en\_in}_X = (\text{tct}_Y - \text{tct}_X < C)$ (capacity C not reached)

- What if there are more than two wires? [Will return to this]

The Composition Theorem: An Instance

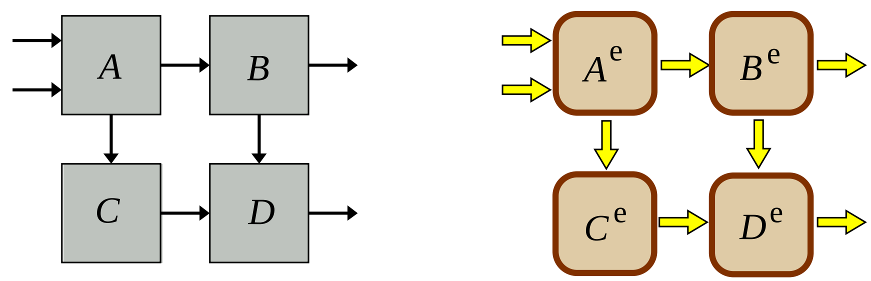
- Two elastic inverters of capacities C_1 and C_2 can make an elastic buffer of capacity $C_1 + C_2$.



- We can check that the combined circuit has the required correctness properties.

General Composition Theorem (Desired)

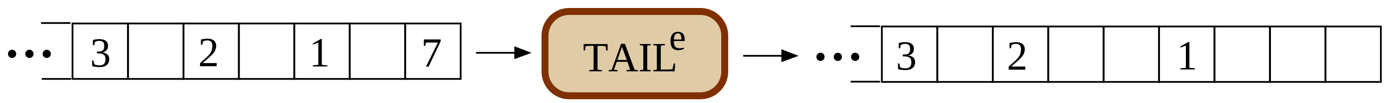
¿Theorem? Suppose the reference system R is obtained from subsystems $R_1, \ldots, R_n$ by joining wires. Suppose also that $R_1^{\mathsf{e}}, \ldots, R_n^{\mathsf{e}}$ are elasticizations of these subsystems. Then if we join the channels of the circuits R_i^{e} following the same pattern in which R is obtained from the R_i, the resulting circuit will be an elasticization of R.



- Proved in [FMCAD'06] for a restricted class of reference systems and a restricted definition of elastic circuits.

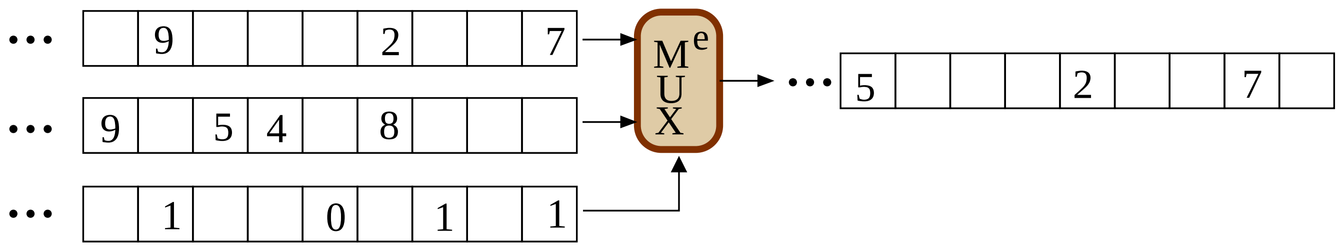
Extended Theory Needed

- Reference systems need not be circuits



  - Token count inside can be negative

- Early evaluation [Cortadella&Kishinevsky, DAC'07]

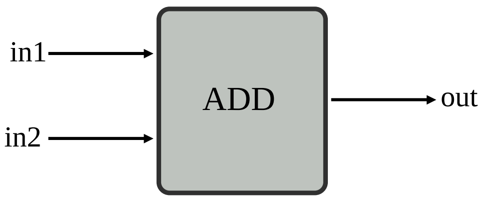


  - Output tokens produced without waiting for irrelevant input tokens
  - Token count on a channel can be negative ("anti-tokens")

Systems Modeled as Stream Transformers

Definition Given a set of wires W, a **W-system** is a set of behaviors, where a behavior is a W-indexed records of streams.

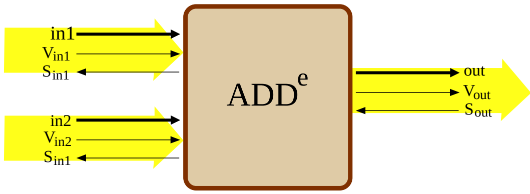
Definition In a **functional system**, $W = I + O$, and output streams depend functionally on the inputs. $(F : [\![I]\!] \to [\![O]\!])$



- $W = \{\text{in1}, \text{in2}, \text{out}\}$, $I = \{\text{in1}, \text{in2}\}$, $O = \{\text{out}\}$
- Behaviors: $\sigma = \langle \sigma.\text{in1}, \sigma.\text{in2}, \sigma.\text{out} \rangle$ where

$$(\forall i = 0, 1, \ldots)\ (\sigma.\text{out}[i] = \sigma.\text{in1}[i] + \sigma.\text{in2}[i])$$

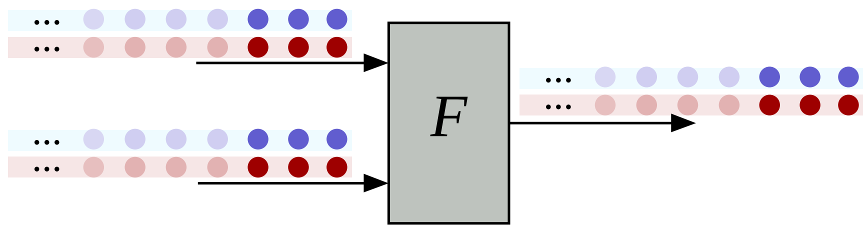
- Ex: $\sigma.\text{in1} = \langle 2, 2, 2, \ldots \rangle$ $\quad \sigma.\text{in2} = \langle 1, 2, 3, \ldots \rangle$
  $\sigma.\text{out} = \langle 3, 4, 5, \ldots \rangle$



- $W = \{\text{in1}, \text{in2}, \text{out}, \text{V}_{\text{in1}}, \text{V}_{\text{in2}}, \text{V}_{\text{out}}, \text{S}_{\text{in1}}, \text{S}_{\text{in2}}, \text{S}_{\text{out}}\}$
- Behaviors: ... (nine streams in each)

Causal Systems (Circuits Abstractly)

Outputs at the first k cycles are determined by inputs at the first k cycles.



Definition A **causal system** is given by a function $F\colon [\![I]\!] \to [\![O]\!]$ satisfying the property

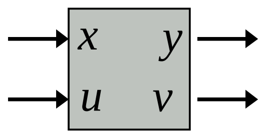
$$\begin{pmatrix} \forall k \geq 0 \\ \forall Y \in O \\ \forall \sigma, \sigma' \in [\![I]\!] \end{pmatrix} \bigwedge_{X \in I} \sigma.X \sim_k \sigma'.X \implies F(\sigma).Y \sim_k F(\sigma').Y$$

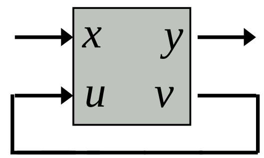
- **Definition** $a \sim_n b$ iff $\mathsf{prefix}(n, a) = \mathsf{prefix}(n, b)$
- Example: $a = \langle 1, 2, 3, 4, 5, \ldots \rangle \qquad b = \langle 1, 2, 7, 4, 5, \ldots \rangle$

$$\therefore\ a \sim_2 b \qquad\qquad \therefore\ a \not\sim_3 b$$

Sequential Dependency

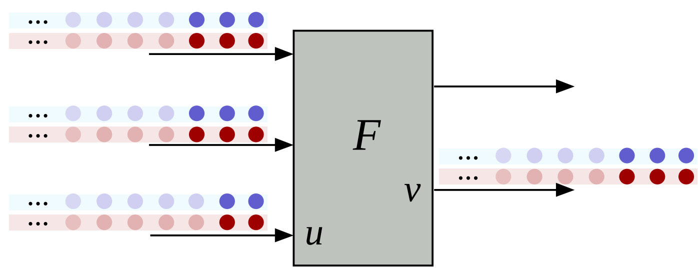
- Feedback: When is it a causal system too?





Definition An input-output pair (u, v) is **sequential** if

$$\left(\begin{array}{c}\forall \sigma, \sigma' \in [\![I]\!] \\ \forall k \geq 0\end{array}\right) \quad \begin{array}{c} \sigma.u \sim_{k-1} \sigma'.u \\ \wedge \\ (\forall x \neq u)\ \sigma.x \sim_k \sigma'.x \end{array} \implies F(\sigma).v \sim_k F(\sigma').v$$



Feedback Lemma If (u, v) is sequential, then the feedback system is causal.

Systems with Specified Delays

Give an integer δ_{XY} for each input-output pair X, Y; let $\delta = \|\delta_{XY}\|$.

Definition A **δ-system**:

$$\left(\begin{array}{c} \forall k \geq 0 \\ \forall Y \in O \\ \forall \sigma, \sigma' \in [\![I]\!] \end{array} \right) \bigwedge_{X \in I} \sigma.X \sim_{k-\delta_{XY}} \sigma'.X \implies F(\sigma).Y \sim_k F(\sigma').Y$$

- δ_{XY} "is" the initial number of tokens between X and Y
- We saw a negative δ_{XY} in the TAIL system

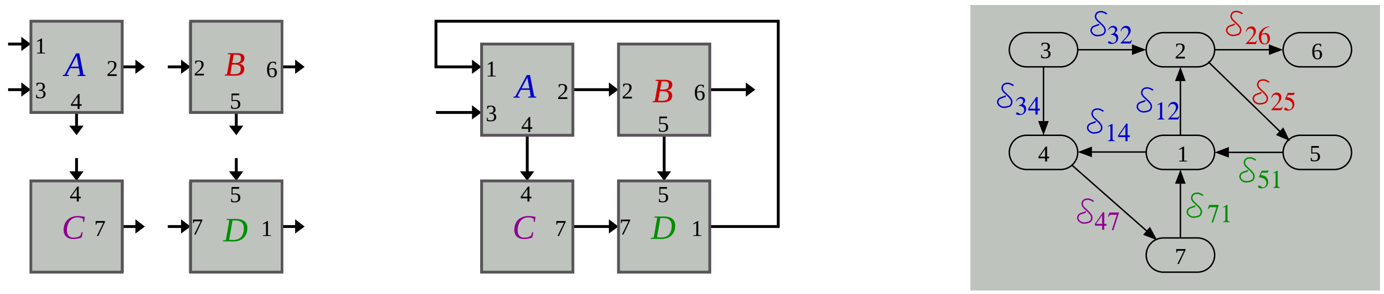
The Feedback Theorem for δ-systems

Theorem Suppose F is a δ-system and $\delta_{UV} > 0$. Then F' is a δ'-system, where

$$\delta'_{XY} = \min(\delta_{XY}, \delta_{XV} + \delta_{UY})$$

for every input-output pair X, Y.

The Cycle Sum Test [Wadge, 1979]



Given a network of δ-systems, draw the graph whose nodes are the wires in the system, and the edges are the input-output pairs of all component systems, each labeled with its delay number.

Theorem If the sum of labels along any closed path is positive, then the network is a legitimate δ-system.

- Improves the combinational loop theorem
- The original Wadge's result is about Kahn networks
- Similar result by Gay&Natarajan (2003) in a categorical model of networks (with positive δ's)

Back To Modeling Elasticity: Token Counts

- Liveness conditions

  $\Box(\Box \text{en\_out}_Y \rightarrow \Diamond V_Y)$ $\Box(\Box \text{en\_in}_X \rightarrow \Diamond \neg S_X)$

- The enabling predicates? —For the inverter we had
  - $\text{en\_out}_Y = (\text{tct}_Y - \text{tct}_X > 0)$ (there is a token inside)
  - $\text{en\_in}_X = (\text{tct}_Y - \text{tct}_X < C)$ (capacity C not reached)

- New notation:
  - δ_{XY} = the initial number of tokens between X and Y
  - tct_{XY} = the current number of tokens between X and Y

∴ Basic equation: $\text{tct}_X + \delta_{XY} = \text{tct}_Y + \text{tct}_{XY}$

∴ en_out_Y must be at least as strong as $(\forall X \in I)(\text{tct}_{XY} > 0)$

The Capacities and Bubble Counts

- There must be a maximum C_{XY} of the number of tokens between X and Y.
- New notation:
  - β_{YX} = the initial amount of free space between X and Y
  - bct_{YX} = the current number of free space between X and Y

∴ Basic equation: $\mathsf{tct}_Y + \beta_{YX} = \mathsf{tct}_X + \mathsf{bct}_{YX}$

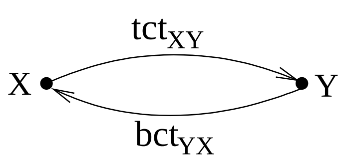
∴ Also: $\mathsf{tct}_{XY} + \mathsf{bct}_{YX} = \delta_{XY} + \beta_{YX} = C_{XY}$

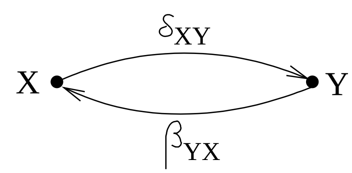
∴ $\mathsf{en\_in}_X$ must be at least as strong as $(\forall Y \in O)(\mathsf{bct}_{YX} > 0)$

The Associated Marked Graph

- Elastic systems should be parametrized by matrices $\delta = \|\delta_{XY}\|$ and $\beta = \|\beta_{YX}\|$

- Marked graph: Vertices are wires and a pair of edges joins every input with every output, labeled by token/bubble counts



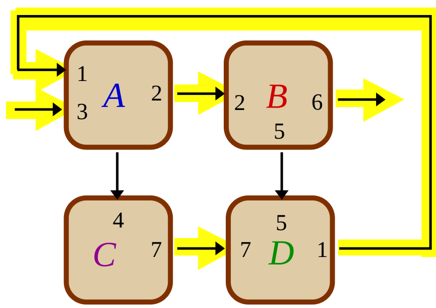


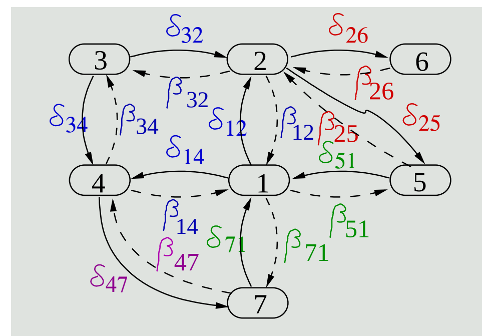
initial marking

- Condition on (δ, β): No non-positive cycles in the initial marking

Composition Theorem (Reformulated)

¿Theorem? Suppose a network E is built by joining channels of elastic systems $E_1, \ldots, E_n$. (Each E_i is (δ_i, β_i)-elastic.) If the Wadge graph that includes all input-ouptut and output-input edges passes the cycle sum test, then E is an elastic system.





- Special case: $n = 1$, and E is obtained from E_1 by a single feedback
  - Sufficient to prove the general case

We Can't Prove It

- Liveness conditions for $E_1, \ldots, E_n$ should imply the same conditions for E
- The minimal enabling conditions
  - $\mathsf{en\_out}_Y \;=\; (\forall X \in I)(\mathsf{tct}_{XY} > 0)$
  - $\mathsf{en\_in}_X \;=\; (\forall Y \in O)(\mathsf{bct}_{YX} > 0)$

  guarantee that at least one channel is always enabled
- Stronger condition for outputs
  - $\mathsf{en\_out}_Y \;=\; (\forall X \in I)(\mathsf{tct}_{XY} > 0) \;\wedge\; (\forall Y' \in O)(\mathsf{tct}_{Y'} \leq \mathsf{tct}_Y)$

  works if the matrix δ is positive
- ☞ Help develop the theory of elastic circuits!